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Massive ghost theories with a line of defects

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Abstract

We study free massive fermionic ghosts, in the presence of an extended line of impurities. The corresponding scattering theory can be formulated by adding to the bulk S-matrix the scattering amplitudes, describing the interactions among the bulk excitations and the defect line (transmission and reflection amplitudes). Explicit expressions for such matrices can be found by solving a bootstrap system of equations (unitarity, crossing and factorization) or, alternatively, relying on a Lagrangian description in terms of Symplectic fermions. In this framework, two distinct defect interactions are proposed (a relevant and a marginal ones), and exact expressions for the correlation functions of the most significant operators in the theory are derived, exploiting the bulk form factors and the matrix elements relative to the defect operator, encoding the entire information about the inhomogeneities.

1 Introduction

After the seminal work by Ghoshal and Zamolodchikov [1] on integrable field theories in the presence of a boundary, a great deal of attention has been devoted to study finite size effects, due especially to their numerous applications to real physical problems. Quantum field theories with extended line of defects generalize these boundary models, introducing new and original features [2, 3, 4, 5].

The presence of impurities can be mimicked by the action of a 'defect' operator, placed along an infinite line in the Euclidean space. In the continuum limit and away from criticality, massive excitations can either participate to bulk scattering processes or interact with the defect. In general, due to the breaking of translational invariance, only reflection and transmission are allowed. Such information can be encoded into a scattering theory enriched by adding to the bulk S-matrix the amplitudes relative to these two new processes. The integrability of the model, originally studied in [2], is guaranteed by imposing the factorization condition which translates into a set of cubic relations called the Reflection-Transmission equations. In particular, it has been showed that, for diagonal bulk scattering, non-trivial solutions for both the reflection and transmission amplitudes can be found only in non interacting bulk systems. In this light, free field theories play a prominent rôle.

Recently, a wide interest has grown around free ghosts in two dimensions[1], due to their relevance to the study of disordered systems, polymer physics, quantum Hall states [9, 10, 11, 12, 13] and above all as an example of the simplest non-unitary/logarithmic conformal field theories [14, 15].

The main purpose of this work is to generalize a previously studied model of free massive fermionic ghosts [16], in order to include the effects of inhomogeneities. In particular, the knowledge of the scattering amplitudes (and the spectrum of bulk excitations), along with general analyticity properties and relativistic invariance, allows to reconstruct thoroughly the off-shell dynamics, by computing exactly correlation functions.

The first step towards the realization of this program involves the derivation of the transmission and reflection amplitudes. One way to compute them consists in solving a bootstrap system of equations (unitarity, crossing and factorization). However, in this peculiar case, the absence of stringent constraints leaves a broad arbitrariness in the choice of the solutions. Fortunately, an alternative description is possible, in terms of the Lagrangian formalism

[1]An exhaustive analysis of the fermionic and bosonic ghosts' conformal field theories, possessing respectively conformal charges $c = -2$ and $c = -1$, can be found in [6, 7, 8].

2 Bootstrap approach

The model we are going to study is that of free massive fermionic ghosts [16] in the presence of an infinite line of impurities, placed[2] at $x = 0$.

The bulk spectrum of the theory is composed of a doublet of free particles A and $\bar{A}$ with mass m, bearing respectively U(1) charges ± 1. Their scattering is ruled, in the bulk, by the S-matrix $S = -1$. Due to the energy conservation, when a particle hits the defect it can be either reflected or transmitted. All the processes involved in the theory can be recast as a set of algebraic equations [2], relying on the algebra of the Faddeev-Zamolodchikov operators. After the usual parameterization of the particle's energy-momentum in terms of the rapidity variable $(e, p) = (m\cosh\theta, m\sinh\theta)$, we associate to excitations of type 'a' the formal operator $A_a(\theta)$ and to the defect line an operator $\mathcal{D}$, playing the rôle of a zero rapidity particle, during the whole time evolution of the system. The commutation relations, associated to the defect algebra, read

$$\begin{aligned} A_a(\theta)\mathcal{D} &= R_a^b(\theta)A_b(-\theta)\mathcal{D} + T_a^b(\theta)\mathcal{D}A_b(\theta)\,, \\ \mathcal{D}A_a(\theta) &= R_a^b(-\theta)\mathcal{D}A_b(-\theta) + T_a^b(-\theta)A_b(\theta)\mathcal{D}\,, \end{aligned} \tag{1}$$

where, in the first equation, $R_a^b(\theta)$ and $T_a^b(\theta)$ denote, respectively, the reflection and transmission amplitudes of an asymptotic particle 'a' entering the defect with rapidity θ, from the left[3]. Consistency of (1) implies the unitarity conditions

$$\begin{aligned} R_a^b(\theta)R_b^c(-\theta) + T_a^b(\theta)T_b^c(-\theta) &= \delta_a^c\,, \\ R_a^b(\theta)T_b^c(-\theta) + T_a^b(\theta)R_b^c(-\theta) &= 0\,. \end{aligned} \tag{2}$$

Crossing relations read

$$\begin{aligned} &\mathcal{C}^{aa''}R_{a''}^b\left(i\frac{\pi}{2} - \theta\right) = S_{a'b'}^{ab}(2\theta)\mathcal{C}^{b'b''}R_{b''}^{a'}\left(i\frac{\pi}{2} + \theta\right)\,, \\ &T_a^b(\theta) = \mathcal{C}^{bb'}T_{b'}^{a'}(i\pi - \theta)\mathcal{C}_{a'a}\,, \end{aligned} \tag{3}$$

with an antisymmetric charge conjugation matrix, such that $\mathcal{C}^2 = -1$. As regards factorization conditions, the main result of [2] guarantees that, for free theories diagonal in the bulk, the Reflection-Transmission equations, descending from integrability, are automatically satisfied.

At this point, solving the bootstrap system of equations (1)-(3), we are able in principle to determine the scattering amplitudes R_a^b and T_a^b. However, a proliferation of solutions

[2]In the following, the infinite line will be identified with the time axis after a rotation in the Minkowski plane.

[3]The second equation, describing the scattering of a particle hitting the defect from the right, is obtained from the first one, after an analytic continuation $\theta \to -\theta$ in the rapidity variable.

occurs, due to the lack of constraints strong enough to fix the reflection and transmission matrices in a closed form. A simplified version of this model (i.e. a purely reflecting theory which coincides with a boundary problem [1]) helps visualizing the situation. Introduce the following parameterization of the reflection matrix components:

$$
\begin{aligned}
R_A^A(\theta) &= f(\theta)R(\theta) \qquad & R_A^{\bar{A}}(\theta) &= g(\theta)R(\theta) \\
R_{\bar{A}}^{\bar{A}}(\theta) &= f'(\theta)R(\theta) \qquad & R_{\bar{A}}^{A}(\theta) &= g'(\theta)R(\theta)\,.
\end{aligned} \tag{4}
$$

Consistency of the bootstrap system gives rise to the conditions

$$
\begin{aligned}
R(\theta)R(-\theta) &= [f(\theta)f(-\theta) + g(\theta)g'(-\theta)]^{-1} \\
R(\theta)R(-\theta) &= [f'(\theta)f'(-\theta) + g'(\theta)g(-\theta)]^{-1}
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
f(\theta)g(-\theta) + g(\theta)f'(-\theta) &= 0 \\
f'(\theta)g'(-\theta) + g'(\theta)f(-\theta) &= 0
\end{aligned} \tag{6}
$$

3 Lagrangian description

To overcome the ambiguities, intrinsically concerned with the bootstrap scenario, the lagrangian approach proves to be an alternative route.

The Euclidean action, describing the bulk dynamics, is that of free massless symplectic fermions[4], supplemented by a mass term

$$
\mathcal{A}_B = \frac{1}{2}\int d^2x\, J_{\alpha\beta}\left(\partial_\mu \Phi^\alpha \partial^\mu \Phi^\beta + m^2\, \Phi^\alpha \Phi^\beta\right)\,. \tag{7}
$$

Φ^α, which are zero dimensional anti-commuting fields (Φ and $\bar{\Phi}$), belong to the same doublet, characterized by mass m, while $J_{\alpha\beta}$ is an antisymmetric tensor. A detailed analysis of the bulk system, including mode expansions of the basic fields, commutation relations and charge conjugation properties, can be found in the Appendix A.

Inhomogeneities affect the bulk physics introducing a Lagrangian density along the impurity line, according to (**??**). A relevant and a marginal interactions will be the object of our study in order to derive explicit expressions for the reflection and transmission amplitudes.

[4] A careful study of the symplectic fermions at the critical point may be found in [7].

3.1 Relevant perturbation

Consider the system described by

$$\mathcal{A} = \mathcal{A}_B + \frac{g}{2} \int d^2x \, \delta(x) \, J_{\alpha\beta} \Phi^\alpha \Phi^\beta \,, \tag{8}$$

where the dimension of the coupling constant g is $[mass]$. The equations of motion read

$$\begin{aligned}(-m^2)\Phi &= g\delta(x)\Phi \\ (-m^2)\bar{\Phi} &= g\delta(x)\bar{\Phi} \,.\end{aligned} \tag{9}$$

It is useful to split the fields into components belonging to the two intervals $x < 0$ and $x > 0$ (after rotation to the Minkowski space)

$$\begin{aligned}\Phi(x,t) &= \theta(x)\Phi_+(x,t) + \theta(-x)\Phi_-(x,t) \\ \bar{\Phi}(x,t) &= \theta(x)\bar{\Phi}_+(x,t) + \theta(-x)\bar{\Phi}_-(x,t) \,,\end{aligned} \tag{10}$$

in order to derive the boundary conditions at $x = 0$, given by

$$\begin{aligned}\Phi_+(0,t) - \Phi_-(0,t) &= 0\,; \\ \partial_x(\Phi_+(0,t) - \Phi_-(0,t)) &= \frac{g}{4}(\Phi_+(0,t) + \Phi_-(0,t))\end{aligned} \tag{11}$$

$$\begin{aligned}\bar{\Phi}_+(0,t) - \bar{\Phi}_-(0,t) &= 0\,; \\ \partial_x(\bar{\Phi}_+(0,t) - \bar{\Phi}_-(0,t)) &= \frac{g}{4}(\bar{\Phi}_+(0,t) + \bar{\Phi}_-(0,t))\,.\end{aligned} \tag{12}$$

The mode expansions (30), in terms of the operators A and $\bar{A}$ which interpolate the bulk excitations, allow us to extract explicitly from (11)-(12) the reflection and transmission amplitudes

$$\begin{pmatrix} A_-(\beta) \\ \bar{A}_-(\beta) \\ A_+(-\beta) \\ \bar{A}_+(-\beta) \end{pmatrix} = \begin{pmatrix} R(\beta,\kappa) & T(\beta,\kappa) \\ T(\beta,\kappa) & R(\beta,\kappa) \end{pmatrix} \begin{pmatrix} A_-(-\beta) \\ \bar{A}_-(-\beta) \\ A_+(\beta) \\ \bar{A}_+(\beta) \end{pmatrix} , \tag{13}$$

with

$$\begin{aligned}R(\beta,\kappa) &= \frac{1}{\sinh\beta + i\kappa} \begin{pmatrix} -i\kappa & 0 \\ 0 & -i\kappa \end{pmatrix}, \\ T(\beta,\kappa) &= \frac{1}{\sinh\beta + i\kappa} \begin{pmatrix} \sinh\beta & 0 \\ 0 & \sinh\beta \end{pmatrix}\end{aligned} \tag{14}$$

and $\kappa = g/4m$. R and T, thus obtained, satisfy crossing and unitarity conditions.

A strong analogy with the free bosonic theory, extensively treated in [2], emerges. A part from a doubling of the matrix elements, the scattering amplitudes coincide. The main features are the occurrence of resonances[5] for $\kappa > 1$ and phenomena of instabilities for $\kappa < -1$, characterized by poles with imaginary part fixed at the value $i\pi/2$, acquiring an increasing real part as κ is further depleted.

In the limit $g \to \infty$ ($\kappa \to \infty$), corresponding to the fixed boundary conditions $\Phi(0,t) = 0$ and $\bar{\Phi}(0,t) = 0$, the defect line acts as a purely reflecting surface. On the contrary, in the high-energy limit $\beta \to \infty$, due to the relevant character of the perturbation, the theory renormalizes to a bulk regime, the impurity line becoming transparent.

3.2 Marginal perturbation

The Euclidean action

$$\mathcal{A} = \mathcal{A}_B - ig \int d^2x\, \delta(x)\, (\Phi\partial_y\Phi + \bar{\Phi}\partial_y\bar{\Phi})\,, \tag{15}$$

where g is a dimensionless coupling constant, describes the effects of a marginal interaction on the defect line. The equations of motion

$$(-m^2)\bar{\Phi} - 2ig\delta(x)\partial\Phi = 0 \tag{16}$$

$$(-m^2)\Phi + 2ig\delta(x)\partial\bar{\Phi} = 0 \tag{17}$$

lead to the following boundary conditions in the Minkowski plane

$$\begin{aligned} \bar{\Phi}_+(0,t) - \bar{\Phi}_-(0,t) &= 0\,; \\ \partial_x(\bar{\Phi}_+(0,t) - \bar{\Phi}_-(0,t)) &= g\,\partial_t\Phi(0,t) \end{aligned} \tag{18}$$

$$\begin{aligned} \Phi_+(0,t) - \Phi_-(0,t) &= 0\,; \\ \partial_x(\Phi_+(0,t) - \Phi_-(0,t)) &= -g\,\partial_t\bar{\Phi}(0,t)\,. \end{aligned} \tag{19}$$

Exploiting again the mode expansions in terms of the operators A and $\bar{A}$, the reflection and transmission matrices assume the form

$$\begin{aligned} R(\beta,\chi) &= \frac{\sin\chi\cosh\beta}{\cosh^2\beta - \cos^2\chi}\begin{pmatrix} -\sin\chi\cosh\beta & -\cos\chi\sinh\beta \\ \cos\chi\sinh\beta & -\sin\chi\cosh\beta \end{pmatrix}, \\ T(\beta,\chi) &= \frac{\cos\chi\sinh\beta}{\cosh^2\beta - \cos^2\chi}\begin{pmatrix} \cos\chi\sinh\beta & -\sin\chi\cosh\beta \\ \sin\chi\cosh\beta & \cos\chi\sinh\beta \end{pmatrix}, \end{aligned} \tag{20}$$

[5] i.e. unstable bound states possessing a real part in the unphysical sheet, which do not appear as asymptotic particles of the theory.

Let us turn the attention on the analytic structure of the reflection and transmission matrices. Since the theory is non-unitary, a mechanism, akin to the one occurring in the scaling Lee-Yang model [18], is expected to take place. In other words, residues, corresponding to poles in the scattering amplitudes, are not supposed to be, a priori, real and positive. This phenomenon is reminiscent of the non-hermitian nature of the Hamiltonian associated to the system[6], and does not contrast with the unitarity requirement (2), preserving the meaning of probability densities[7].

Poles appear both in the reflection and the transmission amplitudes at $\beta = i\chi$ and $\beta = i(\pi - \chi)$, with $\chi \in [0, \pi/2]$. In the case of diagonal matrix elements, the corresponding residues give

$$
\begin{aligned}
R_A^A \simeq R_{\bar{A}}^{\bar{A}} \simeq T_A^A \simeq T_{\bar{A}}^{\bar{A}} &\simeq \frac{i}{2} \cdot \frac{\sin\chi\, \cos\chi}{\beta - i\chi} \\
R_A^A \simeq R_{\bar{A}}^{\bar{A}} \simeq T_A^A \simeq T_{\bar{A}}^{\bar{A}} &\simeq \frac{i}{2} \cdot \frac{-\sin\chi\, \cos\chi}{\beta - i(\pi - \chi)} \,.
\end{aligned}
\tag{21}
$$

Therefore, the pole at $\beta = i\chi$ is associated to a boundary bound state in the direct channel, with positive binding energy $e_b = m \cos\chi$, while the other one lives in the crossed channel. Since $e_b < m$ for every value of the coupling constant, the boundary bound states are always stable and the theory is free of resonances and instabilities of other nature. As regards off-diagonal processes, the residues calculated at $\beta = i\chi$ assume the form

$$
R_A^{\bar{A}} \simeq T_A^{\bar{A}} \simeq \frac{i}{2} \cdot \frac{i\sin\chi\, \cos\chi}{\beta - i\chi} \qquad R_{\bar{A}}^A \simeq T_{\bar{A}}^A \simeq \frac{i}{2} \cdot \frac{-i\sin\chi\, \cos\chi}{\beta - i\chi} \,, \tag{22}
$$

while residues computed in the crossed channel display an overall minus sign. As mentioned before, the additional factor $\pm i$, appearing in the numerator, is a consequence of the anomalous charge conjugation properties of the ghost fields.

Finally, a comment on the marginal nature of the interaction: performing the ultraviolet limit, except for peculiar values of the coupling constant, all the scattering matrices' components remain simultaneously finite.

[6]Non-hermiticity of the Hamiltonian implies, in particular, its left eigenstates $\langle n_L|$ are not simply the adjoints of the right ones $|n_R\rangle$. Since, in addition, the Fock space states are also eigenstates of the charge-conjugation operator with eigenvalues $(\pm i)^N$, N being the particles' number, the relation $\langle n_L| = \langle n_R|\mathcal{C}$ leads to the completeness condition $\sum_n |n_R\rangle\langle n_L| = \sum_n |n_R\rangle\langle n_R|(\pm i)^n$.

[7]Eq. (2), relying only on the assumption that in and out-kets, constructed in terms of the asymptotic particles A and $\bar{A}$, form a basis in the Hilbert space, is insensitive to hermiticity properties of the Hamiltonian.

4 Correlation functions

The problem at the heart of this paper concerns the computation of correlation functions of the local fields $\phi_i(x,t)$, present in the theory.

To realize this idea, in order to fully exploit the knowledge of the bulk physics, it is worth performing a rotation in the Minkowsi plane ($x \to -it$, $t \to ix$), moving the defect line at $t=0$. In this new picture, the Hilbert space of states is the same as in the bulk and the effects of impurities are taken into account by an operator $\mathcal{D}$, placed at $t=0$, which acts on the bulk states. Therefore, correlation functions assume the form [2]

Let us recall here that asymptotic states are composed of neutral pairs $A(\theta)\bar{A}(\beta)$, obtained by acting with the corresponding operators A and $\bar{A}$ on the vacuum $|0\rangle$. Explicit expressions for the bulk Form Factors have been derived in [16], while the simplest matrix elements of the defect operator on the bulk states are

$$\begin{aligned}
\langle A(\theta)|\mathcal{D}|A(\theta')\rangle &= 2\pi\, \hat{T}^{AA}(\theta)\, \delta(\theta-\theta')\,, \\
\langle \bar{A}(\beta)A(\theta)|\mathcal{D}|0\rangle &= 2\pi\, \hat{R}^{A\bar{A}}(\theta)\, \delta(\theta+\beta)\,, \\
\langle 0|\mathcal{D}|A(\theta)\bar{A}(\beta)\rangle &= -2\pi\, \hat{R}^{A\bar{A}}(\theta-i\pi)\, \delta(\beta+\theta-2\pi i)\,.
\end{aligned} \tag{23}$$

In the remaining part of this section, we are going to study correlators of the operator

4.1 ω operator

The simplest correlation function involving ω is the one-point function, defined as[8]

For free boundary conditions, the reflection matrix is trivially zero and the one-point function vanishes. In the case of fixed boundary conditions, instead, $\hat{R}^{A\bar{A}}(\theta) = -1$ and the short distance limit is easily derived

[8] The resolution of the identity explicitly reads

$$1 = \sum_{n=0}^{\infty} \frac{1}{(n!)^2(2\pi)^{2n}} \int_{-\infty}^{+\infty} d\theta_1 ... d\beta_n\, |A(\theta_1), ..., \bar{A}(\beta_n)\rangle\langle \bar{A}(\beta_n), ..., A(\theta_1)| \tag{24}$$

.

Concerning the relevant perturbation, (??) assumes the form

An analogous analysis can be performed for the marginal interaction. The one-point function (??) specializes to

We turn now the attention to the two-point functions involving the operator ω. Two different situations can occur.

Consider the case in which the operators lie on opposite sides of the defect line, i.e. $t_1 < 0$ and $t_2 > 0$. The correlator is given by

Another situation can happen, in which the two ω operators reside on the same half of the Minkowsi plane. Let us consider, for convenience, $t_2 \geq t_1 > 0$ and define $t \equiv t_2 - t_1$, $\bar{t} \equiv t_2 + t_1$, $x \equiv x_2 - x_1$, $r \equiv \sqrt{x^2 + t^2}$. The general expression for the two-point function is

4.2 Disorder operator

Finally, we examine the one-point function of the disorder operator μ[9], which is non-local with respect to the ghost fields. A detailed discussion about such operators in bulk free theories can be found in [20, 21, 22] (ordinary complex fermions and bosons) and [16] (fermionic and bosonic ghost systems). The one-point correlator can be written as follows

Again, free boundary conditions lead to the trivial solution $\mu_0 = 0$.

In the case of fixed boundary conditions, it is possible to recover the leading short-distance behavior of the one-point function, in an exact way. The details of the calculation will be postponed to the Appendix B, while here only the main results will be given. Since the reflection matrix component $\hat{R}^{A\bar{A}}$ is trivially -1, exploiting the theory of Fredholm determinants [23], μ_0 can be recast as

[9] Actually, μ is only a specific example of operator belonging to the widest class of the 'disorder' fields. The analysis concerning the leading behavior of their correlators, which relies on a 'cluster' expansion, is the main purpose of the Appendix C.

As regards the effects produced by the relevant perturbation, (**??**) behaves as

$$\mu_0(t;\kappa) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{1}{\pi}\right)^n \int_{-\infty}^{+\infty} d\theta_1 ... d\theta_n \left[\prod_{k=1}^{n} \frac{\kappa\, e^{-2mt\cosh\theta_k}}{2\,(\cosh\theta_k + \kappa)}\right] |A_n| = \det\left(1 + \frac{1}{\pi} V(t;\kappa)\right), \tag{25}$$

with the kernel

$$V(\theta_i,\theta_j,t;\kappa) = \frac{e(\theta_i,t;\kappa)\, e(\theta_j,t;\kappa)}{2\cosh\frac{\theta_i+\theta_j}{2}}\,,$$
$$e(\theta,t;\kappa) = \sqrt{\frac{\kappa}{\cosh\theta + \kappa}} \cdot e^{-mt\cosh\theta}\,. \tag{26}$$

In the short-distance limit, $|V|^2$ becomes unbounded, the leading singularity being dictated by the fixed boundary conditions' one. Thus we find the same critical exponent as in the previous case.

More interesting is the marginal situation. From general considerations extrapolated from the Ising model [24, 25], the non-universal nature of the marginal interaction is expected to affect the non-local sector of the theory, inducing a critical exponent continuously dependent on the coupling constant. Indeed, μ_0 assumes the form

$$\begin{aligned} \mu_0(t;\chi) &= \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{\sin^2\chi}{\pi}\right)^n \int_{-\infty}^{+\infty} d\theta_1 ... d\theta_n \left[\prod_{k=1}^{n} \frac{\sinh^2\theta_k\, e^{-2mt\cosh\theta_k}}{2\,(\cosh^2\theta_k - \sin^2\chi)}\right] |A_n| = \\ &= \det\left(1 + \frac{\sin^2\chi}{\pi}\, V(t;\chi)\right), \end{aligned} \tag{27}$$

where

$$V(\theta_i,\theta_j,t;\chi) = \frac{e(\theta_i,t;\chi)\, e(\theta_j,t;\chi)}{2\cosh\frac{\theta_i+\theta_j}{2}}\,,$$
$$e(\theta,t;\chi) = \sqrt{\frac{\cosh^2\theta - 1}{\cosh^2\theta - \sin^2\chi}} \cdot e^{-mt\cosh\theta}\,. \tag{28}$$

Repeating an analysis similar to the one carried out for the fixed boundary condition, but this time with a parameter depending on the coupling constant, in front of the kernel in (27), we finally obtain the critical exponent

5 Final remarks

In this paper we have studied the effects induced by a defect interaction on the free theory of massive fermionic ghosts.

Working in the Lagrangian approach, we have dealt with two defect perturbations, respectively, of relevant and marginal nature. Explicit expressions for the reflection and transmission matrices have been derived. A careful analysis of their excitation spectra has pointed out the possibility of resonances and instabilities in the former case, and the occurrence of imaginary residues, relative to poles in the scattering amplitudes, in the latter one. Successively, we turned our attention to the exact computation of correlation functions, involving the most interesting operators in the theory, i.e. ω, local in the ghost fields, and μ, belonging to one of the non-trivial sectors of the model. In the marginal situation, a non-universal behavior in the one-point function of the 'disorder' operator μ has clearly emerged. Finally, the last appendix has been devoted to the analysis of the most general 'disorder' fields μ_α, characterized by non-locality index α. The leading short-distance behavior of their one-point function has been investigated by means of the 'cluster' expansion [26, 27].

It is worth noticing that a delicate point of the present discussion concerns the comparison between the bootstrap approach and the Lagrangian description, in order to derive explicit expressions for the reflection and transmission amplitudes. In the former case, a richness of solutions descends but their physical explanation and 'classification', in terms of a fixed number of parameters related to the bulk S-matrix, results problematic. On the other hand, the Lagrangian approach, though subjected to the strong restriction of dealing only with local interactions, allows for a limited number of solutions, amenable of an easiest control. For instance, besides the defect perturbations already introduced, analyzing other kind of interactions could help identifying new boundary conditions and, possibly, the operator content of the boundary theory.

Finally, we conclude with a remark on the simplified problem of a pure reflecting surface. As hinted at the end of the second section in relation to the free Dirac massive fermions, free theories, derived as limit of interacting ones, admit a richer structure, as it appears clearly in the bootstrap approach. It would be tempting, in this boundary case, to find an interacting theory, if any, behind the fermionic ghost model.

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Appendix A

In this section, some useful results on the bulk system of fermionic ghosts are collected. The action is described by Eq. (7) where the symplectic form $J_{\alpha\beta}$ reads explicitly

$$J_{-+} = -J_{+-} = 1 \ , \ J_{\alpha\gamma}J^{\gamma\beta} = \delta_{\alpha}^{\beta} \, , \tag{29}$$

and the ghost fields $\Phi^{\pm}$, for later convenience, are redefined according to

$$\begin{aligned} \Phi^{+} &\to \Phi \\ \Phi^{-} &\to \bar{\Phi} \, . \end{aligned}$$

The mode expansions for the components $\Phi_{(\pm)}$ and $\bar{\Phi}_{(\pm)}$, previously introduced (10), are

$$\begin{aligned} \Phi_{(\pm)}(x,t) &= \int d\beta \left[\bar{a}_{(\pm)}(\beta) e^{-im(t\cosh\beta - x\sinh\beta)} + a^{\dagger}_{(\pm)}(\beta) e^{im(t\cosh\beta - x\sinh\beta)} \right] \, , \\ \bar{\Phi}_{(\pm)}(x,t) &= \int d\beta \left[-a_{(\pm)}(\beta) e^{-im(t\cosh\beta - x\sinh\beta)} + \bar{a}^{\dagger}_{(\pm)}(\beta) e^{im(t\cosh\beta - x\sinh\beta)} \right] \, , \end{aligned} \tag{30}$$

where the creation and annihilation operators are subjected to the anti-commutation relations

$$\begin{aligned} &\{a_{(\pm)}(\beta), a^{\dagger}_{(\pm)}(\beta')\} = 2\pi\delta(\beta-\beta') \ , \ \{a_{(\pm)}(\beta), a_{(\pm)}(\beta')\} = 0 = \{a^{\dagger}_{(\pm)}(\beta), a^{\dagger}_{(\pm)}(\beta')\}; \\ &\{\bar{a}_{(\pm)}(\beta), \bar{a}^{\dagger}_{(\pm)}(\beta')\} = 2\pi\delta(\beta-\beta') \ , \ \{\bar{a}_{(\pm)}(\beta), \bar{a}_{(\pm)}(\beta')\} = 0 = \{\bar{a}^{\dagger}_{(\pm)}(\beta), \bar{a}^{\dagger}_{(\pm)}(\beta')\} \, . \end{aligned} \tag{31}$$

Charge conjugation implemented on the Fock operators

$$\begin{aligned} \mathcal{C}a(\beta)\mathcal{C}^{-1} &= \bar{a}(\beta) \quad , \quad \mathcal{C}a^{\dagger}(\beta)\mathcal{C}^{-1} = \bar{a}^{\dagger}(\beta) \, ; \\ \mathcal{C}\bar{a}(\beta)\mathcal{C}^{-1} &= -a(\beta) \ , \ \mathcal{C}\bar{a}^{\dagger}(\beta)\mathcal{C}^{-1} = -a^{\dagger}(\beta) \, , \end{aligned} \tag{32}$$

induces the following transformations on the ghost fields $\Phi \to \bar{\Phi}$ and $\bar{\Phi} \to -\Phi$. Finally, it is useful, for notational reasons, to identify the operator creating the bulk excitations with the excitations themselves

$$\begin{aligned} a^{\dagger}(\beta) &\to A(\beta) \, , \\ \bar{a}^{\dagger}(\beta) &\to \bar{A}(\beta) \, . \end{aligned} \tag{33}$$

Appendix B

In this appendix we evaluate the critical exponent of the disorder operator μ, corresponding to the fixed boundary conditions. Let us consider the logarithm of Eq. (**??**)

Appendix C

In this last appendix we discuss generic 'disorder' operators μ_α, which pick up a non-locality phase $e^{\pm 2\pi i\alpha}$, when they are taken around the ghost fields in the Euclidean plane

$$\begin{aligned}
\Phi(z\,e^{2\pi i},\bar{z}\,e^{-2\pi i})\mu_\alpha(0,0) &= e^{2\pi i\alpha}\Phi(z,\bar{z})\mu_\alpha(0,0)\,, \\
\bar{\Phi}(z\,e^{2\pi i},\bar{z}\,e^{-2\pi i})\mu_\alpha(0,0) &= e^{-2\pi i\alpha}\bar{\Phi}(z,\bar{z})\mu_\alpha(0,0)\,.
\end{aligned} \tag{34}$$

In particular, we are interested in deriving the leading short-distance behavior of their one-point function in the case of fixed boundary conditions, in order to perform a comparison with the exact result previously obtained for the specific value $\alpha = \frac{1}{2}$.

The starting point is Eq. (**??**), where the Form Factors $f_n^{1/2}(-\beta_1,...,\beta_n)$ must be replaced by the expression [16]

The key point of the standard 'cluster' expansion is that, since the functions h_n depend only on rapidity differences, they contain a redundant variable. Thus, it is possible, at all orders, to extract the integral

On the other hand, the fermionic ghost model displays a substantial difference. The functions h_n^α depend, by construction, on the sum of rapidities. Thus, only contributions of even order in the series (**??**) admit a redundant variable, finally leading to a logarithmic behavior. The remaining terms, of odd order, provide convergent pieces, useful to reconstruct the normalization constant of the one-point function.

In order to study explicitly the short-distance behavior of μ_0^α, we focus the attention on the second order contribution. All we need to know is

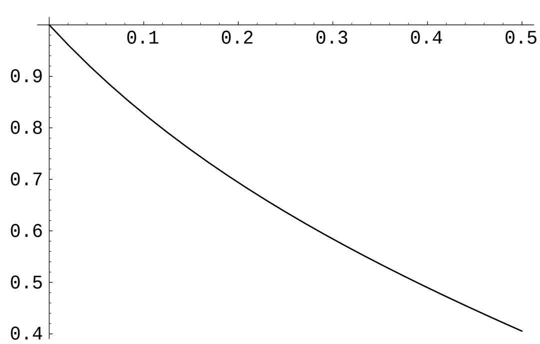

Figure 1: -$\frac{x_\alpha}{\alpha/2}$ as a function of the non-locality index α, for $\alpha \in \left[0, \frac{1}{2}\right]$.

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